A Conformal Finite Difference Time Domain Technique for Modeling Curved Dielectric Surfaces

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***Abstract*—In this paper, we present a simple yet accurate conformal Finite Difference Time Domain (FDTD) technique, which can be used to analyze curved dielectric surfaces. Unlike the existing conformal techniques for handling dielectrics, the present approach utilizes the individual electric field component along the edges of the cell, rather than requiring the calculation of its area or volume, which is partially filled with a dielectric material. The new technique shows good agreement with the results derived by Mode Matching and analytical methods.**

***Index Terms*—CFDTD, dielectric resonator (DR).**

I. Introduction

DIELECTRIC loaded resonators and filters have important applications in many microwave communication devices. Dielectric resonators (DRs) are usually rod-like structures inside a cylindrical enclosure [1], [2]. For the cylindrical resonators commonly used in practical applications, the conventional FDTD algorithm designed for the Cartesian system cannot be employed directly to simulate curved dielectric surfaces [3] in an accurate manner. This is because even with a very fine mesh ($\sim\lambda/25$), the staircasing procedure introduces errors that are significant for narrow-band filter-type applications. Several enhanced FDTD techniques [4]–[6] have been proposed in the literature for modeling curved dielectric surfaces. These approaches employ a weighted volume average concept, which requires the computation of the area and volume of the partially-filled cell. Because these algorithms are based only on the use of the effective dielectric constant of the FDTD cell that is filled with dissimilar materials, they cannot distinguish between cells that have different geometrical properties insofar as the partial filling is concerned, but have the same fill factor.

As mentioned above, the technique for handling curved dielectric surfaces used in this paper is not based on the effective dielectric constant approach. Instead, it makes use of the information on the edges of the cell to devise a field update algorithm, bypassing the area and volume calculations. The numerical results presented in the paper demonstrate that the algorithm produces results that have improved accuracy over existing conformal dielectric techniques.

To validate the proposed approach, we consider two test examples. First, we calculate the resonant frequencies of a cylindrical DR loaded in a rectangular cavity. Next, we investigate a cylindrical DR sandwiched between two parallel PEC plates. Both of these problems have been investigated previously by using other conformal techniques.

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II. FDTD Method for Curved Dielectrics

A. Existing Conformal Dielectric FDTD Algorithms

Typically, existing conformal dielectric FDTD algorithms employ the weighted area or volume average to deal with the cells filled with different materials [4]–[6], as shown in Fig. 1. The corresponding effective dielectric constant used in [5] and [6] is written as

$$\varepsilon_y^{\text{eff}} = \left[\frac{1}{\Delta y}\int_y^{y+\Delta y} \frac{1}{\varepsilon_2 \alpha(y) + \varepsilon_1 (1-\alpha(y))}\, dy\right]^{-1} \tag{1}$$

and

$$\varepsilon_y^{\text{eff}} = \left(S_2(i,j,k)^* \varepsilon_2 + (1 - S_2(i,j,k))^* \varepsilon_1\right)/S(i,j,k) \tag{2}$$

where

Δy cell size along the y direction;

$\alpha(y)$ dielectric surface parameter inside the cells that are filled with dissimilar materials [5];

$S_2(i,j,k)$ partial volume of the cell (i,j,k) that contains these materials [6].

The similar but more complicated calculation may be employed [4].

It is evident that the approaches deal with the average dielectric constant; hence, their use can yield the same effective value for dielectric distributions, even when the geometry of the fillings are different (see Fig. 2). Another fact to bear in mind is that the above approaches require complicated mesh generation, which is not always convenient to implement.

B. Present Conformal Dielectric FDTD Method

The present conformal dielectric algorithm utilizes a linear average concept as shown in Fig. 3, and effective dielectric constants, $\varepsilon_x^{\text{eff}}(i,j,k)$ and $\varepsilon_y^{\text{eff}}(i,j,k)$ as defined below

$$\varepsilon_x^{\text{eff}}(i,j,k) = \left(\Delta x_2(i,j,k)^* \varepsilon_2 + (\Delta x - \Delta x_2(i,j,k))^* \varepsilon_1\right) / \Delta x(i,j,k) \tag{3}$$

$$\varepsilon_y^{\text{eff}}(i,j,k) = \left(\Delta y_2(i,j,k)^* \varepsilon_2 + (\Delta y - \Delta y_2(i,j,k))^* \varepsilon_1\right) / \Delta y(i,j,k). \tag{4}$$

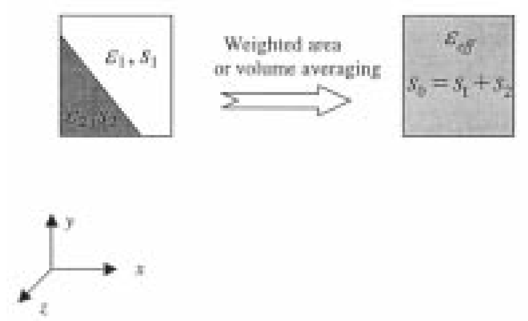

Fig. 1. Conformal dielectric techniques [4, 5, 6].

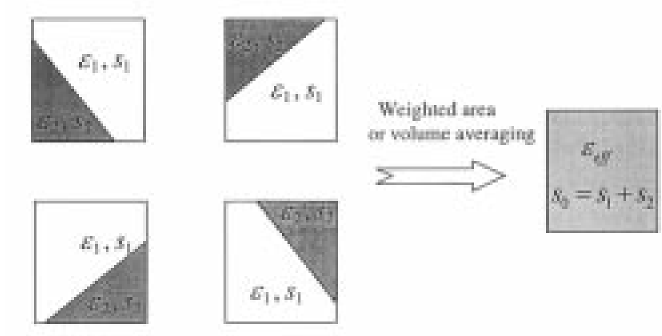

Fig. 2. Illustrative examples showing that different material distributions lead to the same effective dielectric constant.

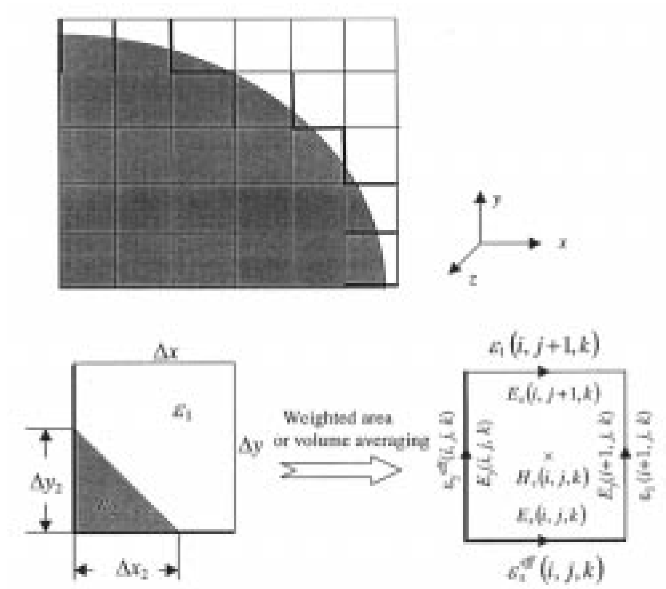

Fig. 3. Linear weighted dielectric constant averaging procedure used in the present conformal dielectric FDTD scheme. (a) Intersection between FDTD mesh and curved dielectric surface. (b) Original problem. (c) Equivalent problem.

We note from Fig. 3 that the edges $E_x(i,\, j+1,\, k)$ and $E_y(i+1,\, j,\, k)$ do not penetrate the dielectric; hence, we use the dielectric constant ε_1, as opposed to the effective dielectric constant employed in the existing conformal techniques, when dealing with them. The update equations for the electric and magnetic fields are thus written as

$$\begin{aligned} E_x^{n+1}(i,\, j,\, k) &= E_x^n(i,\, j,\, k) + \frac{\Delta t}{\varepsilon_x^{\text{eff}}(i,\, j,\, k) * \Delta y} \\ &\quad \cdot \left\{ H_z^{n+1/2}(i,\, j,\, k) - H_z^{n-1/2}(i,\, j-1,\, k) \right\} \\ &\quad - \frac{\Delta t}{\varepsilon_x^{\text{eff}}(i,\, j,\, k) * \Delta z} \\ &\quad \cdot \left\{ H_y^{n+1/2}(i,\, j,\, k) - H_y^{n-1/2}(i,\, j,\, k-1) \right\} \end{aligned} \tag{5}$$

$$\begin{aligned} E_x^{n+1}(i,\, j+1,\, k) &= E_x^n(i,\, j+1,\, k) + \frac{\Delta t}{\varepsilon_1(i,\, j+1,\, k) * \Delta y} \\ &\quad \cdot \left\{ H_z^{n+1/2}(i,\, j+1,\, k) - H_z^{n-1/2}(i,\, j,\, k) \right\} \\ &\quad - \frac{\Delta t}{\varepsilon_1(i,\, j+1,\, k) * \Delta z} \\ &\quad \cdot \left\{ H_y^{n+1/2}(i,\, j+1,\, k) - H_y^{n-1/2}(i,\, j+1,\, k-1) \right\} \end{aligned} \tag{6}$$

where Δy and Δz are the step sizes along the y- and z-directions, respectively. Note that we do use the effective dielectric constant in (5) because the edge on which $E_x(i,\, j,\, k)$ lies does penetrate the dielectric. Note further that the update equation (6) has a form which is identical to that used in the conventional FDTD algorithm because $E_x(i,\, j+1,\, k)$ is located entirely within a single dielectric region, whose permittivity is ε_1. Similar update equations can also be written for other electric field components. It is useful to point out that the present technique can accommodate the case where each of the six sides of an FDTD cell has a different effective dielectric constant; hence, it is possible to apply it to a cell filled with more than two different types of materials.

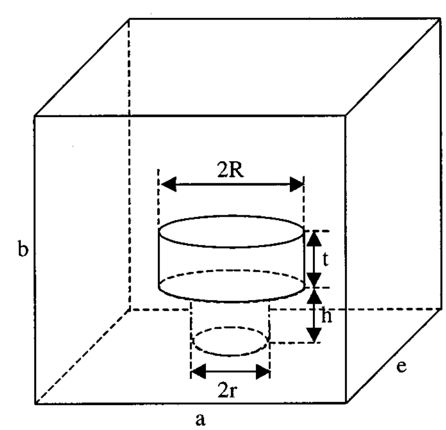

Fig. 4. Dielectric resonator comprising a cylindrical rod in a rectangular cavity. The pedestal below is assumed to be free space in this work.

III. Numerical Results

To validate the proposed conformal dielectric approach, we use it to compute the resonant frequencies of a rectangular cavity loaded with a cylindrical dielectric rod (Fig. 4) as well as a cylindrical DR sandwiched between two parallel PEC plates. For both of these examples, the time step was taken to be

$$\Delta t = \frac{0.995}{c\sqrt{\left(\frac{1}{\Delta x}\right)^2 + \left(\frac{1}{\Delta y}\right)^2 + \left(\frac{1}{\Delta z}\right)^2}}. \quad (7)$$

We note from (7) that we do not have to compromise the Courant condition in this algorithm.

Returning to the dielectric-loaded cavity problem in Fig. 4, we choose, for the sake of facilitating the comparison with previously published results, the relative dielectric constant of the cylindrical rod to be 38 and assume that the pedestal below this rod has a dielectric constant of 1, in accordance with the specifications given in [1]. Next, we generate a nonuniform mesh via the mesh generation software available in [7]. In the simulation of the geometry 1 in Table I, the entire domain includes $48 \times 44 \times 47$ cells, the Δx and Δy are chosen to be 0.519 112 mm inside the dielectric rod, 0.549 275 mm and 0.563 033 mm outside the dielectric rod in the x and y directions, respectively, and the Δz is taken to be 0.537 307 mm, 0.5537 mm, and 0.536 58 mm below, inside, and above the DR, respectively. For the geometry 2 in Table I, the entire domain includes $48 \times 46 \times 47$ cells, the Δx and Δy are chosen to be 0.514 72 mm inside the dielectric rod, 0.564 24 mm and 0.488 95 mm outside the dielectric rod in the x and y directions, respectively, and Δz is taken to be 0.537 308 mm, 0.531 09 mm, and 0.546 65 mm below, inside, and above the DR, respectively. For the geometry 3 in Table I, the entire domain includes $50 \times 46 \times 49$ cells, Δx and Δy are chosen to be 0.509 947 mm inside the dielectric rod, 0.514 35 mm and 0.517 525 mm outside the dielectric rod in the x and y directions, respectively, and Δz is taken to be 0.4989 mm, 0.5355 mm, and 0.5213 mm below, inside, and above the DR, respectively. The simulation was run for 40 000 time steps and the results are shown in Table I, from which we observe that the results from the present scheme agree well with both the Mode Matching values and the measured data. Since the DRs are designed to be inherently narrow-band filters, an accurate estimate of their resonant frequencies is very important for design purposes.

Next, we turn to the second test problem, *viz.*, the computation of resonant frequencies of a cylindrical dielectric rod sandwiched between two parallel PEC plates [5], [6]. The height, radius, and relative dielectric constant of the cylindrical DR are 4.6 mm, 5.25 mm, and 38, respectively. The mesh size is the same as that employed in two previous works [5], [6] that have dealt with the same problem. A six-layer PML [7] is used to truncate the boundaries in the x- and y-directions. The resonant frequencies of the two higher order modes, *viz.*, (HEM_{131} and HEM_{411}), for which the results given in both [5] and [6] deviate noticeably from the theoretical values, are shown in Table II below, along with those derived by using the present scheme.

TABLE I
COMPARISON OF DIFFERENT METHODS FOR A DIELECTRIC ROD IN A RECTANGULAR CAVITY

Size (inch)		Resonant Frequencies (GHz)			
2R	t	Present	Kaneda's [5]	Mode Matching[1]	Measured [1]
0.654	0.218	4.388	4.40	4.3880	4.382
0.689	0.230	4.163	4.17	4.1605	4.153
0.757	0.253	3.725	3.78	3.721	3.777

TABLE II
COMPARISON OF DIFFERENT METHODS FOR A DR

	HEM_{131}	HEM_{411}
Theoretical	9.499	10.579
Literature [5]	9.47	10.56
Literature [6]	9.469	10.56
Present	9.495	10.580

It is evident from the above table that the results obtained via the present conformal dielectric scheme are in very good agreement with the theoretical values.

IV. CONCLUSION

A simple yet accurate scheme to model curved dielectric surfaces in the context of FDTD has been introduced in this paper. The new updating scheme does not require area or volume calculations, and is convenient to apply without the burden of calculating the truncated cell areas and volumes which are partially filled with a dielectric material. Hence, the mesh generation for this conformal technique is quite simple. A more general average formulation may be employed to calculate the effective dielectric constant for large difference between the two dielectric constants.

REFERENCES

[1] X.-P. Liang and K. A. Zakim, "Modeling of cylindrical dielectric resonators in rectangular waveguides and cavity," *IEEE Trans. Microwave Theory Tech.*, vol. 41, no. 12, pp. 2174–2181, Dec. 1993.

[2] S. J. Fiedziusko, "Dual-mode dielectric resonator loaded cavity filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT–30, no. 9, pp. 1311–1316, Sept. 1982.

[3] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Trans. Antennas Propagat.*, vol. AP-14, pp. 302–307, May 1966.

[4] M. Celuch-Marcysiak and W. K. Gwarek, "Higher order modeling of media surfaces for enhanced FDTD analysis of microwave circuits," in *Proc. 24th European Microwave Conf.*, vol. 2, Cannes, France, 1994, pp. 1530–1535.

[5] N. Kaneda, B. Houshm, and T. Itoh, "FDTD analysis of dielectric resonators with curved surfaces," *IEEE Trans. Microwave Theory Tech.*, vol. 45, no. 9, pp. 1645–1649, Sept. 1997.

[6] S. Dey and R. Mittra, "A conformal finite-difference time-domain technique for modeling cylindrical dielectric resonators," *IEEE Trans. Microwave Theory Tech.*, vol. 47, no. 9, pp. 1737–1739, Sept. 1999.

[7] W. Yu and R. Mittra, "A conformal FDTD software package modeling antennas and microstrip circuit components," *IEEE Antennas Propagat. Mag.*, vol. 42, no. 5, pp. 28–39, Oct. 2000.